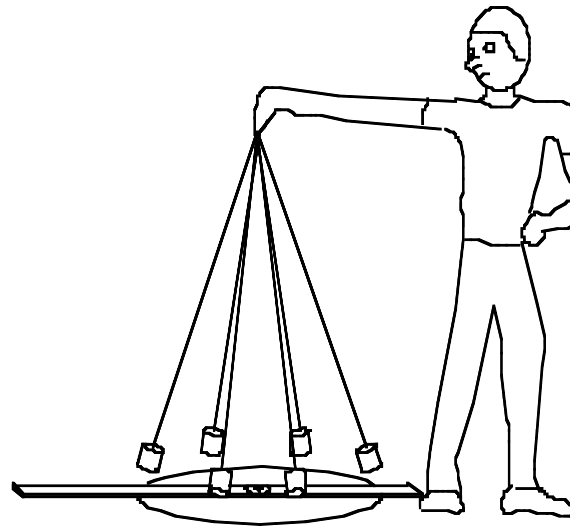
The Conical Pendulum for Centripetal Acceleration

A simple lab requiring modest equipment can be used to verify the traditional expression for centripetal acceleration. It requires a pendulum of about a meter and one half of length with a bob of mass of ½ to 1 kg. A meter stick, a stopwatch, a spring scale and perhaps a washer placed on the meter stick to mark the center of the circle on the floor.

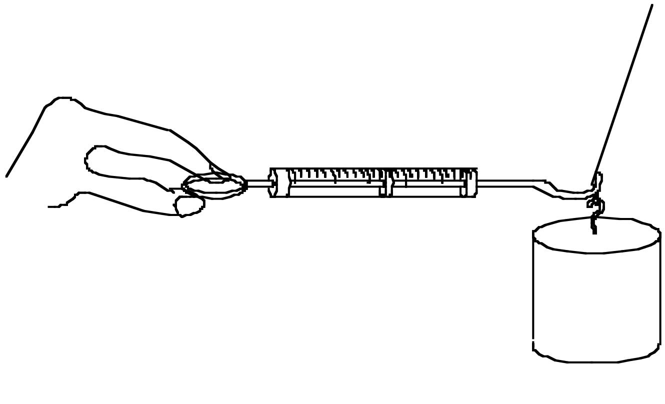
The illustration on the right shows how the pendulum weight can be held and set into rotation in a horizontal circular path. It takes a little patience but when the circular path is established, another observer on the floor will observe the point on the meter stick on either end of the path. Then, with a stopwatch the time will be taken for, say, 10 rotations. With this the radius of the circle can be determined. The circumference of the circle divided by the time of a single rotation will determine the tangential velocity of the weight



The above experiment can be repeated for several different circle diameters. (Students probably do not expect that all different circles with the same pendulum length will have the same period. Let them discover this.)

With the radius and computed tangential velocity the centripetal acceleration can be easily computed using the traditional relation: $\mathbf{a_c = v_t^2 / r}$

Calculating the centripetal acceleration with **F = ma**



Using a spring scale, pull the pendulum weight out to one of the positions where the mass was while moving in a circle. With the measured force divided by the mass, the centripetal acceleration can be computed. This measurement can be repeated for each of the previously measured circles and the accelerations compared.

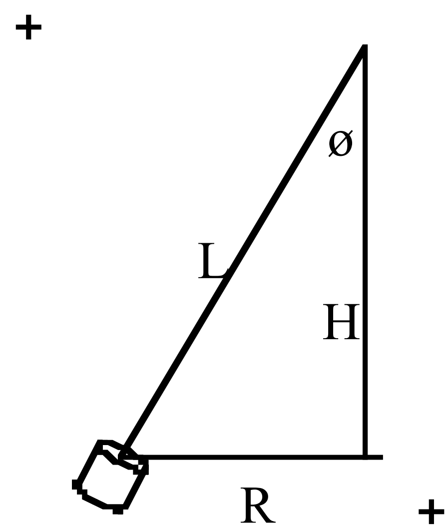
Using trigonometry to compute centripetal acceleration

If your students have trig, the above force measurement is not necessary. Simply by measuring the length of the pendulum and the corresponding circle radius, the centripetal acceleration can be computed.

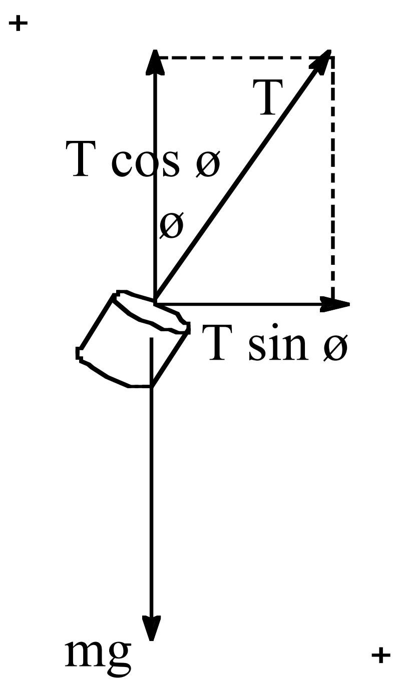
The illustration on the right shows that the angle the pendulum string makes with the vertical is: Ø

It can also be seen that the sin and cos of this angle can be easily found:

Sin Ø = R/L and Cos Ø = H/L



Now let us look at the forces on the bob.



From the diagram on the left we can see the forces acting on the pendulum bob as it swings in a circle. Since there is no vertical acceleration, the vertical component of the tension in the string, T, will equal the weight of the swinging bob. That is:

$$T \cos \phi = mg \text{ or } T = mg/\cos \phi$$

The horizontal component of the tension provides the centripetal force:

$F_c = T \sin \phi$ and from the above $= mg \dfrac{\sin \phi}{\cos \phi}$

Since $a_c = F_c/m$ and $\sin \phi/\cos \phi = \tan \phi$

Centripetal acceleration $a_c = g \tan \phi$

If, during the experiment H was not measured, the Pythagorean theorem could be used to find H and then tan ø would be R/H.