The following are expanded discussions of the basic material to be presented in the first session of NPTW. An important objective of this paper is to keep the concepts clear and simple and to attempt to insure maximum understanding before formulas are presented.

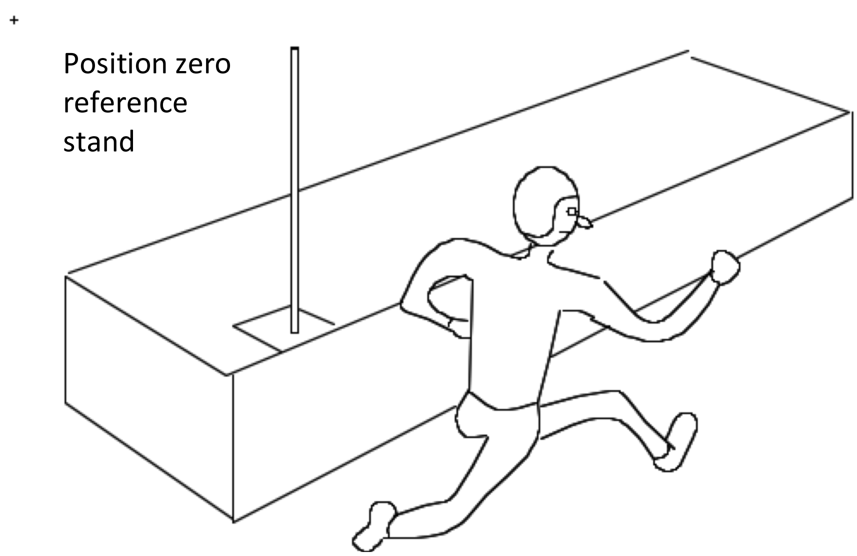
Kinematics

Observing motion and the meaning of position, velocity and acceleration.
Using specific examples, perhaps by even moving around yourself, have students understand the meaning of position (measured in units of length some distance from a specific point), velocity (measured in units of length per time in a specific direction) and acceleration (measured in units of length per time per time or length/time^2 in a specific direction.)

When the actual meanings of position, velocity and acceleration are understood, perhaps mathematical definitions can be given: (These are definitions and not formulas.)
If "s" is used for position and "Δs" for change in position (often called displacement), velocity and acceleration can be defined: $v = \Delta s/\Delta t$ (or lim $\Delta t \rightarrow 0$ of $\Delta s/\Delta t$) and $a = \Delta v/\Delta t$ (or lim$\rightarrow 0$ of $\Delta v/\Delta t$). These shorthand definitions of velocity and acceleration, if understood by the students, will be quite helpful in later discussions. Discussions of limits are not necessary but the ideas of slopes of graphs representing these concepts will be helpful.

Running out data for graphs of motion

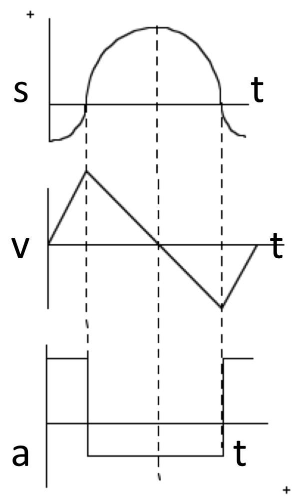
Avoid making kinematics an exclusively mathematical and formula plugging discussion. Running out actual examples of motion by you or students will help make the motion more real and understandable.



Initially use simple motion examples and then move to more complex examples discussing when and in what direction your velocity and acceleration is. When sketching graphs of motion, place position on top, velocity immediately below it and acceleration on the bottom.

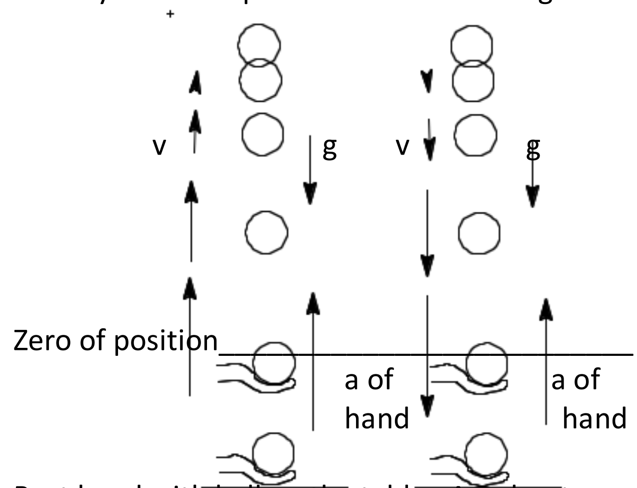
A simple graph problem that is often confusing is to define a vertical coordinate system at some point above the lecture table and toss a ball upward by accelerating it upward for a short period of time, let it go upward and return to your hand and slow it to stop in about the same distance you accelerated it upward. Have the students sketch a position, velocity and acceleration vs. time for the ball during the entire flight each directly below one another. (Suggested solution immediately to the right, Illustration of process, to the far right.)

Suggested sketch solution



The above suggested sketches assume all accelerations are uniform. It may help students if velocity is considered first.

Velocity zero at top but acceleration still g.



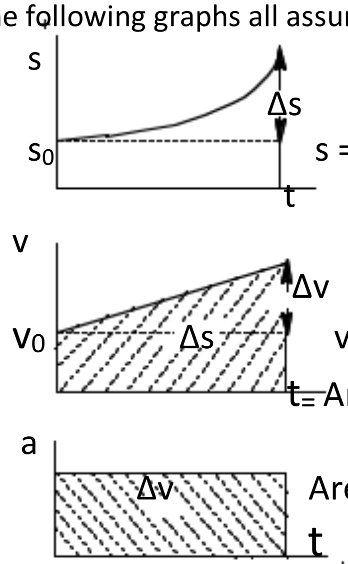
Rest hand with ball on the table. Accelerate upward and release and allow ball to rise while accelerating downward by gravity. When the ball returns to hand at nearly the same position as release point, again apply upward acceleration as originally done upward.

Using basic graphs to derive the kinematics formulas

When sketching graphs of motion, place the position vs. time above the velocity vs. time graph and the velocity vs. time graph above the acceleration vs. time graph. Illustrate and discuss how the slope of the graph above equals the value on the graph below, and how the area of the graph below equals the change in value on the graph above. Show that these follow from the definition of velocity ($v= \Delta s/\Delta t$ or $\Delta s=v\Delta t$) and definition of acceleration ($a=\Delta v/\Delta t$ or $\Delta v= a\Delta t$)

After students appreciate the meaning of graphs of motion, use basic graphs to derive the usual kinematics formulas. This will emphasize learning the meaning of the formulas before simply plugging into them.

The following graphs all assume uniform acceleration



$s = 1/2at^2 + v_0t + s_0$

$v= \Delta v + v_0 = at + v_0$

Area under vt graph=Δs

Area under at graph =Δv

On the left are illustrated position, velocity and acceleration vs. time graphs directly above one another. Notice that the position vs. time graph starts with some slope and an initial value. The initial slope is because there was an initial velocity on the graph below. Also, the area under the entire graph of velocity vs. time will yield the change in position on the above graph. Help your students to understand these slope area concepts follow from the definitions of velocity and acceleration. Stress also, these "formulas" are only for uniform acceleration.

Vectors and basic Trigonometry

Although the NGSS do not advise teaching vectors, many teachers feel they should be taught in physics and are essential if they will be teaching AP. Also as NGSS suggests, consideration of only straight -line kinematics, students will only need to recognize plus and minus direction of motion but projectile and circular motion make little sense without vectors. Even without trig, sketching vectors using rulers and protractors can help students to come to an understanding how scalar math and vector math differ. (Another problem with NGSS is their insistence to confine all discussions to one dimension. This failure to consider the real world of two and three dimensions can be limiting to student understanding.)

A nice simple introduction to vectors is to ask the students: “What is 3 plus 4?” When they answer 7, next suggest how far would you have moved from your starting point if you moved 3 meters north and 4 meters east? Hopefully this will lead to a discussion of situations that are not in a straight line. It should also suggest a need for a type of addition different from scalar addition.

> Define vector addition as: “Place the vectors from point to tail and the vector from the tail of the first to the point of the last is the vector sum.”

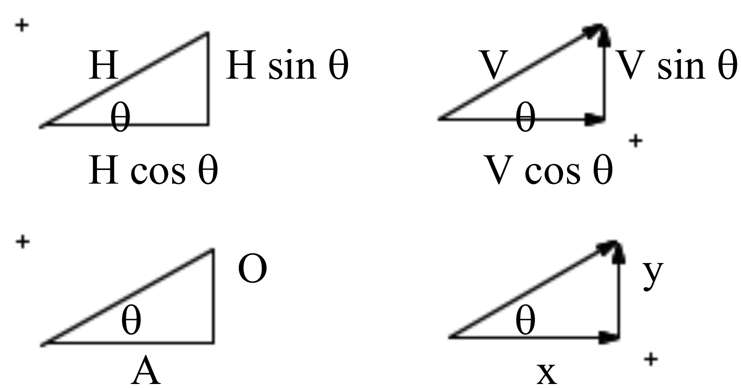
A great introductory exercise in vector addition is to write out a treasure map type of problem with assorted distances and directions to be paced out to find the treasure. Students can use rulers and protractors to find the treasure and learn that this is simply an exercise in vector addition.

A little basic trig can be very helpful in all of physics. Students are often introduced to trig before they reach physics (often in a Geometry class) but a quick and simple review with emphasis on quick use of the trig functions could be helpful.

Suggestion for introduction of simple right angle trigonometry in a physics class:

Using right triangles, define sine, cosine and tangent. After defining the functions stress how, if you know the length of the hypotenuse, you can quickly compute the length of the side opposite the angle (H sin θ) and the length of the side adjacent to the angle (H cos θ) The idea is to recognize how to compute the opposite side and adjacent side immediately. This will then lead to quick computations of the components of a vector

t



If finding the angle is required,

θ = arc tan O/A θ= arc tan y/x

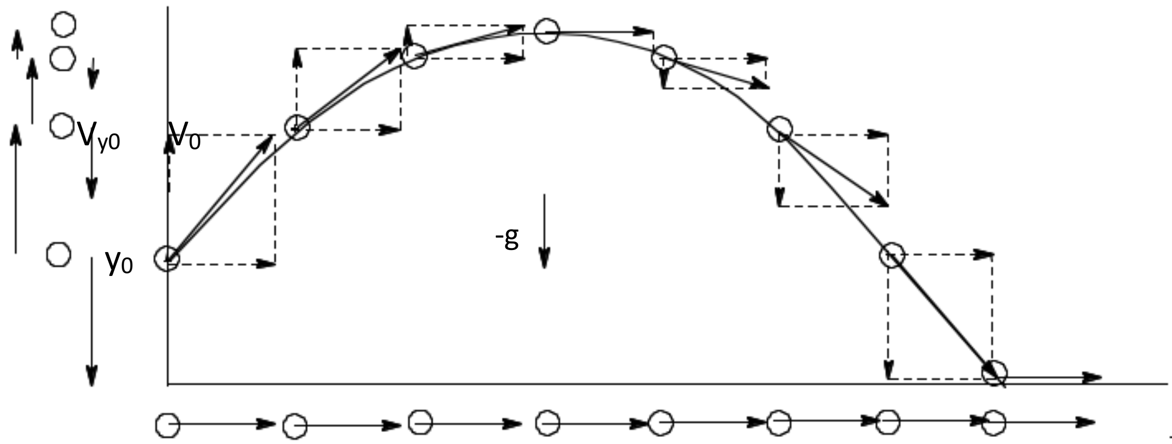
Projectile Motion

The key to understanding projectile motion is to appreciate the actual motion of the projectile at any time is the vector sum of a vertical accelerated motion, and a horizontal motion at constant velocity. Once the projectile is released, the acceleration is always the constant downward acceleration of gravity.

Vertical Motion.

$y=1/2(-g)t^2 +V_{y0} +y_0$

$V_y = (-g)t + V_{y0}$

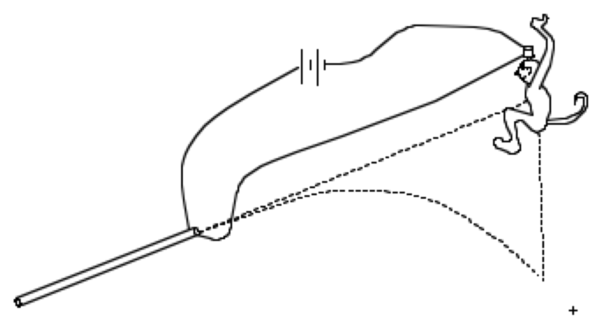


Horizontal motion $x=V_x t$

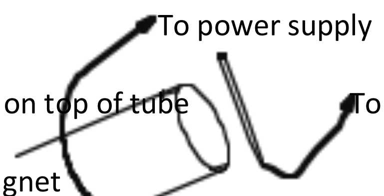
The Monkey and Hunter demonstration

A spectacular demonstration that can help to emphasize the basics of projectile motion is often called the "Monkey and Hunter demonstration". Those who take issue with the name can call it something else.

The story describes a hunter who aims directly at a monkey in a tree and the monkey plans to release his grasp on the tree as soon as the gun is fired thinking the bullet will fly over his head. However, the same gravity that pulls the monkey down will also pull the bullet down, actually insuring a hit. The apparatus for this demonstration can be purchased or with some skill and time, it can be



The gun consists of a metal tube with an inside diameter of about ½ " that will freely accommodate a ½"steel ball. A simple switch (illustrated on the left) at the end of the tube is in series with a power supply and an electromagnet to hold up the target. The gun is firmly clamped and aimed at the target. The ball can be blown with sufficient strength to hit the target before both hit the floor.



Ball slams the vertical wire, separating it from tube and breaking the circuit.

Newton's Laws of Motion

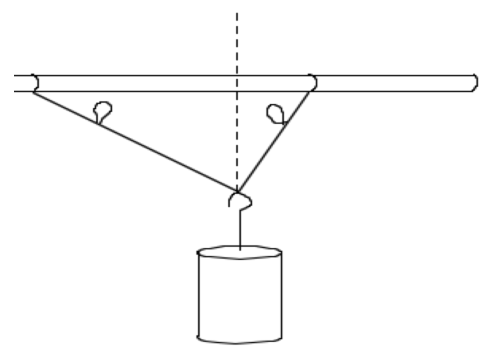
Statics, force vs. torque (The following may be best for a 12^{th} grade High School course.)

Consider introducing "Statics" before "Dynamics". The conditions for objects to be stationary are not trivial yet can help students to appreciate how forces and torques must add to zero on objects at rest, or moving at constant velocity. The two conditions for static equilibrium are:

1. **The sum of all the forces acting on a stationary object must add to zero.**
2. **The sum of all torques acting on a stationary object must add to zero.**

Using the first condition, analyze situations where forces act on a "point object" like considering the forces in strings (tensions) pulling in several directions from a point.

A simple yet effective demonstration to illustrate the first law of static equilibrium is to tie a string to support a 1 kg mass in such a way that either end of the string is tied to a horizontal support such that the angles the strings make with the vertical are 30 and 60 degrees. (Makes calculation easy.) Ask the students: What would a force scale read if inserted into the loops you have placed in the strings and pulled in the direction of the string? (Assume the 1 kg mass weighs about 10 nt. Students often guess that the longer string has the larger force.) Use vectors to compute the solution and test by measuring. Change the angles for further discussion.

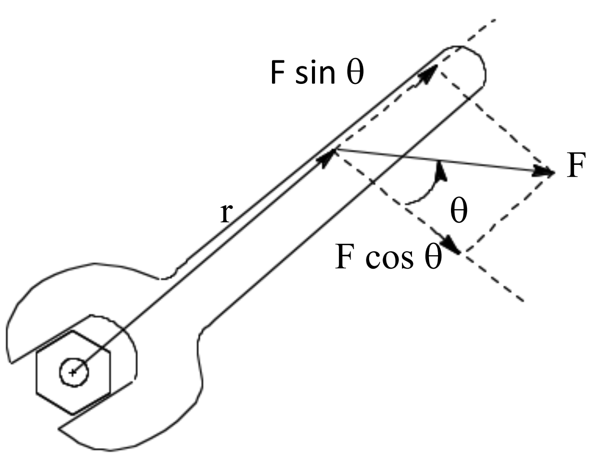


Naturally, the second condition requires understanding the definition of torque (or "the moment of a force"). If we define "moment arm" as the distance from a point to the application of a force, torque can be defined as follows:

Torque is force perpendicular times the momentum arm. (Note, this is the same as moment arm perpendicular times force.)

$$\Gamma = F_{perp}\, r \quad \text{or} \quad \Gamma = r_{perp}\, F \quad \text{or} \quad \Gamma = r \times F$$

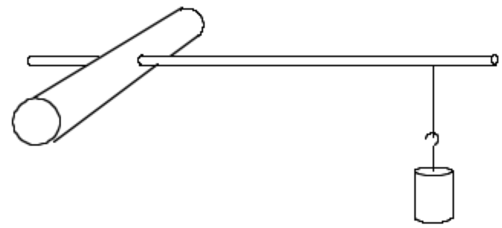
(The use of the vector cross product would be for APC. The order is important.)

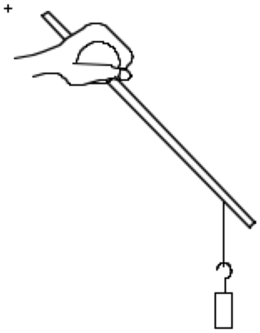


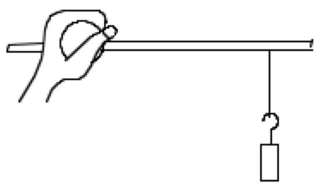
Students might better understand the concept of torque in reference to using a wrench. In the illustration on the left, the point of rotation is the center of the nut, the moment arm, r, is the distance from the point of rotation to the application of the force, F. Since torque is only the component of the force perpendicular to the moment arm, only F cos θ applies (or if the angle of the force were taken from the line of the wrench, then sin and cos would be exchanged in the illustration. Ask the students: "Where and with what angle would they apply the force to get the greatest torque?"

A simple torque demonstrator will help students to understand the definition of torque.

The illustration on the right is of a "torque demonstrator." A large diameter dowel about 6" long has a hole drilled in its center to allow a smaller dowel to be slipped into this hole. The hole should allow the smaller dowel to be slid back and forth yet be held fixed. A small weight is tied near the end of the small dowel producing a downward force to cause the







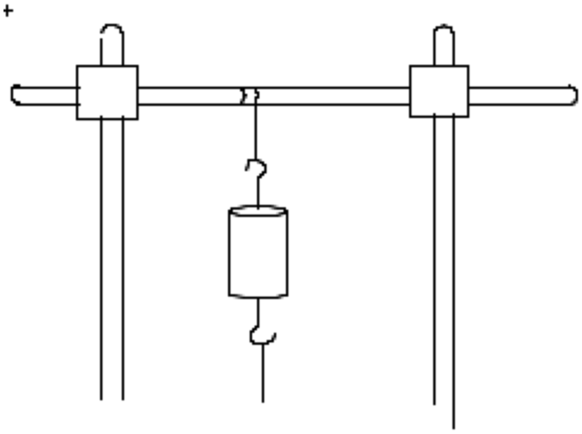
As shown on the left, first hold the large dowel with the weight downward. When the weight is all the way down, no torque is required but as it is rotated upward, more and more torque is required since the force is forming a more perpendicular angle with the moment arm.

As well as rotating the torque demonstrator to show the angle relationship, slide the small dowel in and out of the larger dowel to show how the length of the momentum arm influences the torque.

Dynamics and Newton's laws of motion
Newton's three laws of motion are very important for all levels of physics instruction.

I Bodies at rest will remain at rest and bodies in motion will continue in uniform straight-line motion unless acted upon by an outside force.

An easy but impressive demonstration often called the "inertia ball" is shown on the left. A ball is not needed and a one -kilogram mass with hooks on top and bottom will work well. Light -weight string (of equal strength off of the same roll) like cotton kite string is attached above and below the mass as shown. Use a substantial support since you will be jerking hard on the lower string. Ask the class: "If I pull hard on the lower string, which string will break?" Let a student who answers: "The upper string," explain why. Compliment the student on a logical explanation and then jerk hard and break the bottom string. The subsequent discussion of inertia should follow.



The above demonstration can be enhanced to help students with the concept of experimentation and "The Scientific Method" by saying "I plan to repeat the experiment I just did in exactly the same way. Now which string will break?" It is amazing how many will say: "The top string!" Repeat as many times as you wish breaking the bottom string. (Naturally, retying the bottom string may be required.) Breaking the top string is harder. Use a very slow and constant pull.

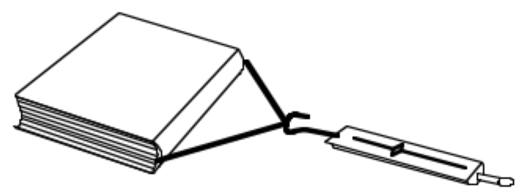
Why it is important to stress that movement is a straight-line is required when introducing Newton's first law? The reason is historical.

Galileo also had a concept of natural motion but he felt that objects would continue to move in a direction parallel to the Earth's surface, or even the sun's surface for planets. This "circular inertia" avoided the need, as Newton recognized, for a force to hold the moon in orbit. That a force is required to change an object's direction of motion is an important addition to the concept of inertia. This is easy to demonstrate using a ball rolling along the floor and hitting it with a stick at right angles to its velocity to change its direction of motion. Newton understood this well and it became the initial thought that led to the development of the law of gravity.

II The usual statement of Newton's second law can be boiled down to: F = ma.

Stress that this is not mass times velocity but mass times change in velocity or acceleration.

A demonstration or lab exercise can be done that involves little equipment and several of the same sized books that most classrooms have on hand, Pass a string through the book as shown and pull it with a force balance, First pull it at constant velocity to establish the force of moving friction and then pull it harder to make it accelerate. The difference between the two forces is the "net" or "unbalanced" force



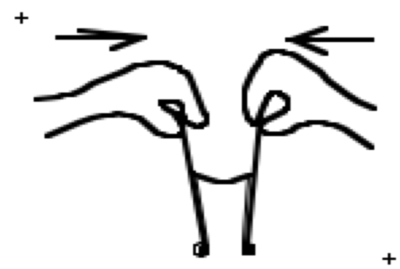
As a lab exercise the students can be provided with several equal books that can be called "unit masses". (The balance can be use to measure the weight of a single book, m=W/g, to calculate the actual mass if desired.) Established a measured distance to use with a stopwatch to calculate the actual acceleration. Stacking books on top of the first book will vary the mass. The resulting calculation of net force, mass and acceleration should give a crude verification of Newton's second law with lots of error to discuss. Using waxed paper and other materials under the bottom book should lead to a nice discussion of the coefficient and force of friction.

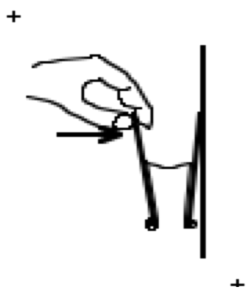
III For every action (force) there is an equal and opposite reaction (force).

Stress that the action force is on one object and the reaction force is on another object. This can be better understood by saying: "If A acts on B then B acts on A." For example, when an object is freely falling above the surface of the earth, what is the action force on the object and where is the reaction force? The action is the earth pulling on the object and the reaction would be the object pulling back on the earth.

The following discussion of Newton's third law has been found useful with elementary school students but high school students often confuse the concepts illustrated. The popular misconception is that only living things and people can exert forces but it is difficult to appreciate that when you push on a wall, it always pushes back with an equal and opposite force.

The first part of the exercise involves a simple binder clip. Have one student press on one side and another press on the other side. Discussion and experimentation should lead to the fact that either student must push as hard in the opposite direction to open the clip.





After establishing that equal and opposite forces are required to open the binder clip, have a single student press the clip against a wall to open it. It should be obvious that the wall presses back. On the right is shown the usual way to open the clip and it should also be obvious that equal and opposite forces are required. The same idea can be used with pulling forces and rubber bands.

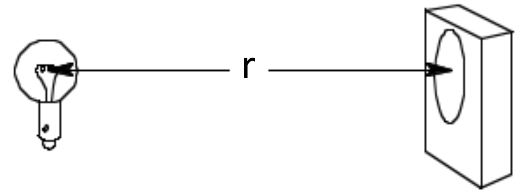
Gravity and Newton's Law of universal gravitation. (The inverse square law.)

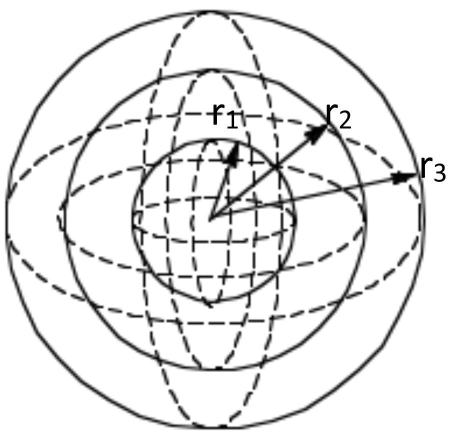
Newton's Law of Gravitation (as well as Coulomb's Law) seems to be a popular example of the inverse square law of force. Stress that in the gravitational law the masses are "gravitational masses" and in Newton's second law, the mass is "inertial mass." The fact that these two "different" masses can be cancelled in assorted problems is because we use the same units for the two "different" masses. That they are the "same" and lead to large and small masses falling at the same rate was a puzzle to Newton.

It can be instructive to ask the class: "Why do large rocks and small rocks fall with the same rate of acceleration?" This can be discussed by stressing that the large rock has a large weight (related to its gravitational mass) and a large mass (its inertial mass.) The mathematical argument could be: $GmM/r^2 = ma$, when you cancel m, on the left side of the equation it is gravitational mass and on the right side of the equation it is inertial mass. That they are measured in the same units makes the math right but still, they are different concepts. Einstein cleared this up.

Since the inverse square law is so important, a good lab or demonstration would be to investigate how light from a small filament light source varies with distance from the source. The geometry of the surface area of a sphere together with light intensity being power per unit area can be used to give a mathematical explanation of the inverse square law.

Using a clear light bulb with a small filament it will be observed that as the bulb and meter are separated, the intensity of the light measured by the meter will vary as the inverse square of the distance between them. Make sure little or no light reflects from light objects (such as your face or hands) back into the meter.



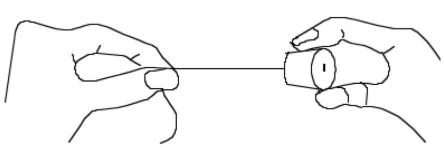


Since the inverse square law is used so frequently, it might be well to help the students appreciate that it is simply the consequence of a small or spherical source sending any kind of energy equally in all directions. This means the energy will spread out uniformly over the surface of a sphere. (Think of a short explosive burst spreading out into three-dimensional space.) Since the energy is spread uniformly over the surface of the sphere and the surface area of the sphere varies as the square of the radius, the <u>intensity</u> of the energy (energy per unit area) will vary inversely as the square of the distance from the source. This works for sound, light, gravitational or electric fields, etc. Ask your students how they think light would vary with distance from a long cylindrical tube like a fluorescent tube. What about an extended plane of light?

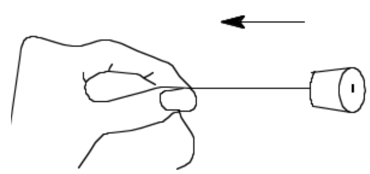
Circular Motion and Satellite dynamics

Change in velocity means change in <u>direction</u> as well as change in <u>speed.</u>
Since acceleration is introduced for straight-line motion, it can be difficult to appreciate that a <u>constant speed</u> changing direction is <u>also</u> acceleration. Now that Newton's second law has connected <u>force</u> and <u>acceleration</u>, discuss the force that is required to move an object in a circle and insist that this force must also cause acceleration.

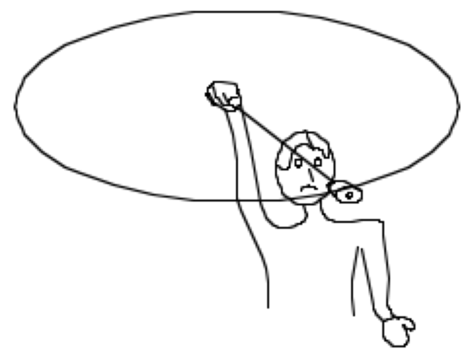
A very useful and simple demonstration device is a rubber stopper (#6 or so) tied to a long string. This device can be used to demonstrate circular motion, pendulums, etc. and should be kept on hand to illustrate things that might come up. Below it is used to help students understand the direction of centripetal acceleration.



As shown on the right, use two hands to hold the stopper and pull the string straight. Ask students: "How could I keep the string horizontal and straight with only one hand?" Hopefully acceleration in the direction shown on the left would be a conclusion. Also discuss if horizontal is even possible.

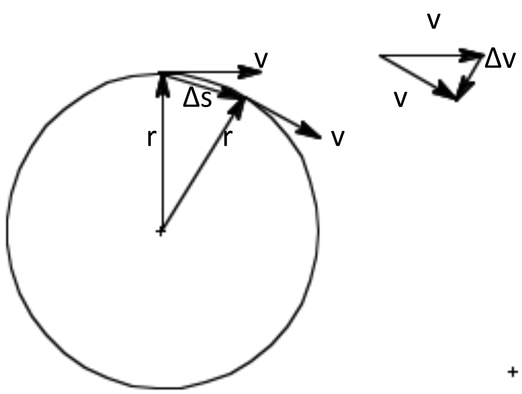


Now twirl the stopper holding on to the string over your head and insist the direction of the force acting on the stopper must be inward or "centripetal". The next step would be to dispel the misconception that there is a force outward on the stopper. As you swirl it in a circle say you intend to release the string when the stopper is aligned directly at a student in the class. "Will the stopper hit her/him?" When you release it, the stopper will continue in the direction of the <u>velocity</u> it was moving. (Be sure nothing is in that direction that might break. The rubber stopper will be OK).



Derivation of the centripetal acceleration expression $a=v^2/r$

Using vector concepts, a not too involved derivation of the centripetal acceleration relationship can be done:



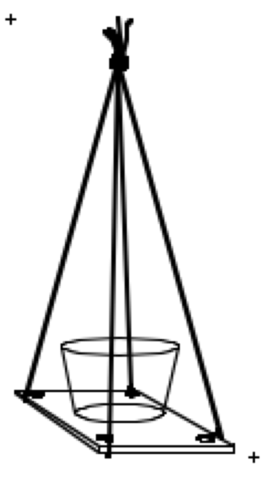
The illustration on the right shows an object moving around a circle at constant speed, v, at two different points on the circle of radius r. The vector diagram to the left shows the velocity vectors at the two different times. Since these velocity vectors are always at right angles to the radius vectors, they sweep out the same angle and form an isosceles triangle similar to the isosceles triangle of the two radius vectors. This means $\Delta s/r = \Delta v/v$. From the definition of acceleration: $a = \Delta v/\Delta t$, and from the similar triangles, $\Delta v = v\,\Delta s/r$. Replacing Δv into the acceleration expression gives; $a = (v/r)(\Delta s/\Delta t)$

However, Δs/Δt <u>is</u> our definition of velocity, v, so the equation becomes; $\mathbf{a = v^2/r}$

This derivation actually deals with average velocity and acceleration but if your students appreciate the definition of velocity involving limits, the result applies for every instant the object moves on the edge of the circle at constant speed. This expression for centripetal acceleration will be used often throughout a physics course.

A popular demonstration of centripetal force is to swing a bucket of water overhead at a sufficient speed to hold the water in the bucket. The following demonstration done before or after this bucket of water demonstration should help students to understand the basic physics.

Using a clear drinking cup hold a small ball in the upside down cup with your finger as shown on the left above. Remove your finger and the ball falls. Ask students: "What must I do to keep the ball in the cup without holding it with my finger?" When they suggest moving it downward, move it downward at constant velocity. (Again the ball falls out.) Additional discussion should lead to the conclusion that it must be <u>accelerated</u> downward with the acceleration equal to, or greater than, the acceleration of gravity. This discussion should help to avoid the often used fictitious: "centrifugal force."



A nice demonstration device that can easily be constructed is shown on the left. Some call it a "Greek waiters tray". Drill four holes in a thin piece of wood about 4 inches on a side. Attach four strings as shown and tie them in a single knot about a meter above. When a clear cup of water is placed on the platform, it is possible to move the knot end of the platform in all sorts of different motions and the water will not spill out of the cup. It is particularly spectacular to swing the cup and platform overhead and the water will not spill from the cup. Before swinging make sure you have clear space overhead or the result will be catastrophic.

Newton's law of universal gravitation and satellite orbits

The radius of the orbits of earth satellites can be solved using Newton's law of universal gravitation and the centripetal acceleration relationship.

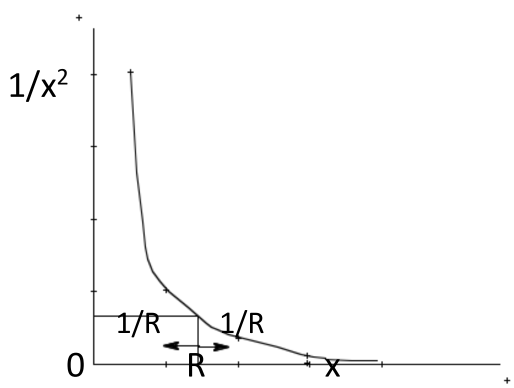
Force of gravity = GMm/r^2. Centripetal force = mv^2/r. Equating and solving for r:

$r = GM/v^2$ Where M is the mass of the earth in this case.

Calculating the distance of a geo-synchronous satellite from the earth is a problem that should interest everyone in this age of extensive dependence upon these devices. Express the velocity of the satellite in orbit in the above equation in terms of the circumference of the orbit and the period, T. required to circle the earth once: $v= 2\pi r/T$. Substituting this in the above equation will give the radius in terms of the period. For a geo-synchronous orbit, set the period at one day in seconds. Remember, this will be the radius of the orbit and not the distance from the surface of the earth.

Energy in an inverse square law. (This discussion may be too mathematical for NPTW and requires knowledge of work and energy that will be discussed later in this paper.)

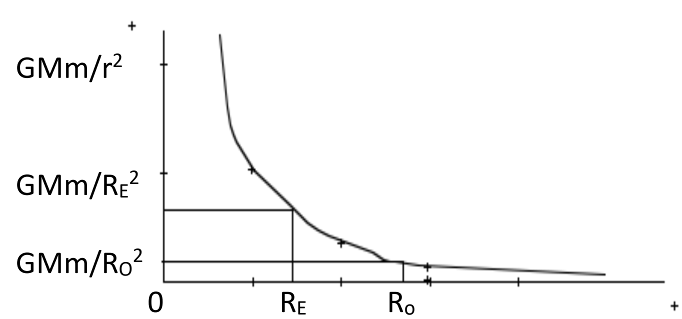
A graphical fact about the function $y = 1/x^2$ that can help compute the energy to launch a satellite into orbit is that the area under the graph of the function from some value to infinity equals the area under the rectangle from this value back to the coordinate axis. (Illustrated below.) Without calculus this graph fact can be memorized and will be useful in the following discussions. In an APC class the same result can be obtained from integrating the function $y=x^{-2}$ from infinity to a specific x. $\int_R^\infty x^{-2}dx = 1/R$



When you are plotting an inverse square function against its argument, the area under the graph from a specific value (R on the graph on the left) to infinity will always equal the area under the graph from this specific point (R), back to zero. When computing the energy to carry a mass from some specific distance, R, to infinity, it will also equal the area of the rectangle from the point R back to the vertical axis. Hence from GMm/r^2 we can find the energy to lift a mass, m, from R to infinity and it will equal GMm/R

When placing a satellite in orbit, the total energy required will be the energy required to lift it to the orbital distance from the earth, plus the kinetic energy required to get it moving fast enough to stay in orbit.

The graph on the right is the force of gravity plotted against the distance from the center of the earth. The first rectangle of GMm/R_E^2 by R_E would be the energy required to carry the mass m to infinity from the surface of the earth. The area of the rectangle GMm/R_O^2 would be the energy required to carry the mass from the orbital distance to infinity. The difference between these two areas is the work required to



lift the mass from the surface of the earth to its orbital distance, or its potential energy. Recognizing that the velocity of a satellite must be exactly the value in which the centripetal force on the satellite at a particular orbital radius equals the force of gravity, the kinetic energy of the satellite can be computed. KE of orbit = ½ mv^2. From the centripetal acceleration expression, the force to hold the mass in orbit at radius R_O is $GMm/R_O^2 = mv^2/R_O$. Notice that if we multiply both sides of the equation by $1/2R_O$ we get ½ mv^2 = ½ GMm/R_O.

Putting together the potential energy to lift the mass from the surface of the earth to orbital distance and the kinetic energy required to keep it in orbit we get:

Total energy = PE + KE = ($GMm/R_E - GMm/R_O$) + $1/2GMm/R_O$ = GMm/R_E -$1/2GMm/R_O$

Another interesting concept, escape velocity, which is the <u>velocity</u> required to supply sufficient kinetic energy to carry an object from the surface of the earth to infinity. This can now be calculated by equating $1/2mv^2$ to GMm/R_E. (Note, this velocity would be the escape velocity from the earth, the satellite would still be trapped by the sun.)

Momentum

Impulse and momentum

Linear momentum is one of physics' major conserved quantities. Unlike energy, it has only one form, mass times velocity. That is, <u>momentum **p** = m**v**</u>. It is always conserved in all sorts of situations like collisions, rocket launches, jumping in the air, everything.

A good way to introduce momentum conservation is by using two carts on a track with magnets repelling one another on one end and Velcro on the other end. (Details below under collisions.) Use these carts to show elastic and inelastic collisions of the same and different mass carts. Repeatedly stress that in <u>all</u> situations, momentum is conserved.

Next, run across the front of the room and quickly stop yourself against a wall. Ask: " Where did the momentum go?" The subsequent discussion should lead to an understanding that the momentum was transferred from the earth to you as you accelerated positively and was transferred back to the earth as you accelerated negatively against the wall. The huge mass of the earth moved at such a minutely small velocity that it was not noticed. Now explain what would happen if you were on a skate board and accelerated in one direction. Discuss what would happen to the skate board which is a small mass doing the same thing the earth did.

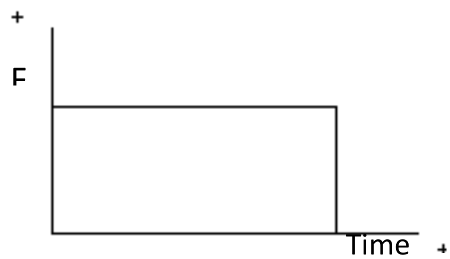
Impulse is defined as unbalanced Force times the time it acts. That is, <u>Impulse = F Δt.</u> The impulse-momentum theorem states that impulse = change in momentum. (Often this is cited as more like Newton actually stated his second law.)

Graphs of force vs. time

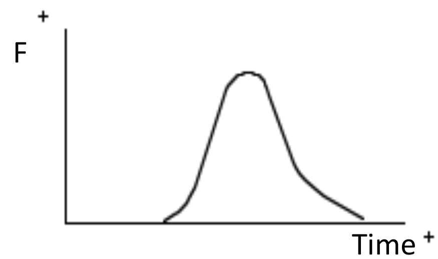
When we discus Newton's second law, the forces are often held constant. In impulse situations, particularly during collisions, the force of interaction does not remain constant. A more general definition of impulse is: the area under a force vs. time graph. (In APC this is the same thing as: Impulse = $\int \mathbf{F}\, dt$)

$$\text{Impulse } \mathbf{F}\Delta t = \Delta(m\mathbf{v}) \text{ change in momentum}$$

Frequently it makes sense to plot force vs. time graphs to compute the area under the graph for impulse.



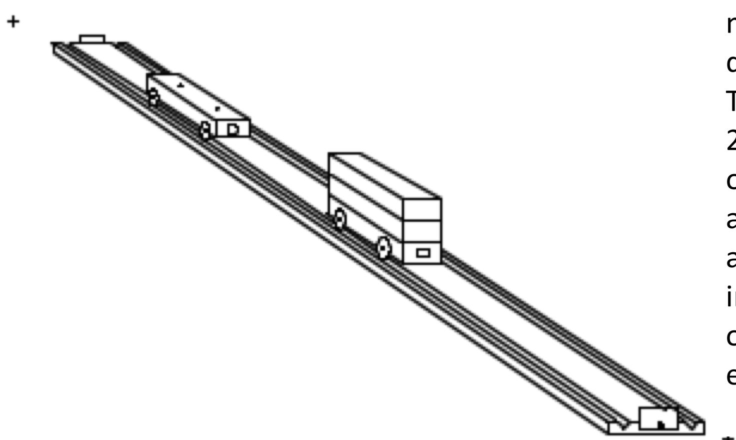
On the right is an impulse graph for a simple constant force. On the right is a more typical graph of impulse during a collision. The area under the graph is impulse.



Show your students that the basic units of impulse and momentum are the same. For some reason, there is no single name for the units of impulse or momentum. Impulse is Newton seconds and momentum is kilogram-meter per second. Of course, these two different names are the same.

Collisions

The track and carts mentioned above are perfect for demonstrating elastic and inelastic collisions.



The track and carts illustrated on the left are made from inexpensive materials and are distributed every other year at the NPTW. The carts are made from identical pieces of 2X4 lumber, the track is from a 4 foot piece of 1 x 6 with V shaped grooves. lined with aluminum angle. The wheels are from Pitsco and two large opposing magnets are inserted in one end of the cart with Velcro on the other end for inelastic collisions. The entire apparatus costs less than $20.

An intuitive discussion of how momentum is conserved can involve Newton's 3rd law and the time the forces act on each of the two colliding objects. Since the forces must be the same but in opposite directions and the times the forces act on each object must be the same, the loss of momentum by one (the negative impulse) must exactly equal the gain in momentum (the positive impulse) on the other.

Stress that in all sorts of collisions, momentum is always conserved. Even when the cart slows down after the collision, the momentum is not lost but is transferred to the earth.

Energy

Work transfers energy, contrast this with impulse transfers momentum

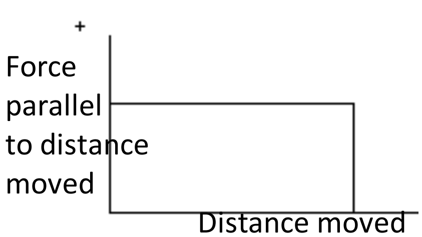
Students may better understand the difference between force and work if they consider just pushing on a wall without its moving vs. pushing on a car by hand to get it started which moves faster and faster while moving it. In which case do you get winded? When the wall does not move, no work is done by the force. After a long time pushing the wall you might get tired but not like when you push the car that starts moving. When the car moves through a greater distance, you get winded because the energy in your body is being transferred to the kinetic energy of the car.

Work involves force and distance. Work transfers energy. Impulse involves force and time. Impulse transfers momentum. Also, work is a scaler and impulse is a vector.

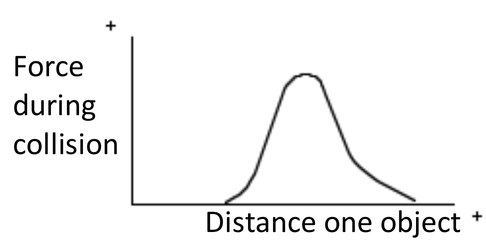
Definitions of work
Definitions of work may differ depending upon the possible sophistication desired. The following are all correct and are listed in the order of sophistication and possible application:

Work is force times distance.
Work is the component of the force in the direction of motion times displacement.
Work is the area under a force parallel vs. position graph.
For use in AP Physics C: Work = $\int_{s1}^{s2} Fdotd\,t$ (The "dot" means the vector dot product)

(If you earlier taught torque it might be well to point out that the units of work and torque are the same but they are different concepts. Since with torque the force is always perpendicular to the moment arm (distance) and with work the force is always parallel to the distance. Even though the basic units are the same, a joule of work is not the same as a meter newton of torque.)



The graph on the left could represent an object moving across a friction surface at constant velocity. On the right is the force between two objects during a collision.



In introductory physics classes, graphs of the component of the force parallel to the direction of motion vs. the distance through which the force acts can be very useful, particularly when discussing situations in which the force is not constant. Non-constant forces are almost always the case when energy is transferred during collisions.

Potential energy and Kinetic energy

When a force accelerates an object, it gives it kinetic energy. When a force acts against a conservative force, it stores potential energy.

(Conservative forces, for example gravity or the force of a spring, can be understood as forces that "remember". That is, if the force passes through a point and later returns to the same point, the force will be the same. Friction is not a conservative force since the work done is turned to heat and later returning to the same point will not experience the same force, often being in a different direction.)

The expression for kinetic energy is so frequently used; it should be derived and memorized. Starting with the basic definition: Work = Fs. If F is constant and freely accelerates an object starting from rest, $v^2 = 2as$ or $s = v^2/2a$ (This comes from eliminating the t in the two simple kinematics expressions: $s = 1/2at^2$ and $v = at$)

Replacing s in the work definition and recognizing that F = ma, will give:

$$\text{Work} = ma\, v^2/2a = 1/2mv^2 = \text{Kinetic Energy}$$

When students know the expression for kinetic energy, they will often forget what it means. Consider the following question:

A force of 30 newtons acts upward to lift 10 nt weight from rest vertically upward a height 2 meters.

1. Find the potential energy of the weight the instant after lifting it 2 meters.
2. Find the kinetic energy of the weight the instant after lifting it 2 meters.

It is surprising how often students will use kinematics to find the velocity of the weight and the height it is lifted so they can plug into mgh and $1/2mv^2$ to find the answers. The work against a conservative force (potential energy) or 10nt x 2 m = 20 joule and the work of an unbalance force (kinetic energy) 20nt x 2m = 40 joule will give the immediate answers.

Heat and Thermodynamics

Heat vs. Temperature

Heat and temperature are often confused. Temperature has an elaborate operational definition but for our use it should be sufficient to state that Temperature is a measure of the average random kinetic energy <u>per molecule</u> in an object whereas Heat is a measure of the <u>total</u> random kinetic and potential energy of molecules in an object. Actually, as it would be incorrect to speak of the work in an object, it is also incorrect to speak of the heat in an object. Heat and work relate to the transfer of energy rather than something that an object contains.

Specific heat and Heat Capacity

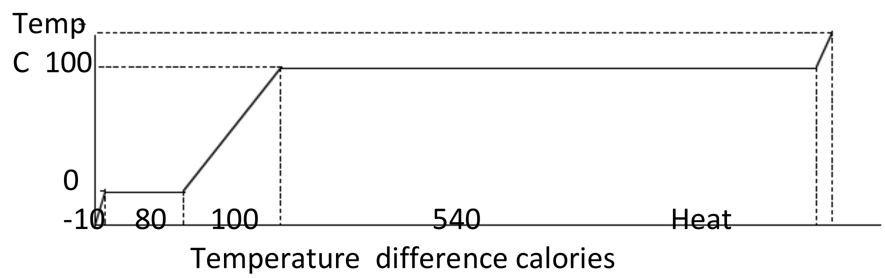
A simple relationship that is often discussed in Chemistry addresses the heat required to change the temperature of an object. If the phase does not change, the amount of heat transferred to an object will equal a constant times the mass of the object times its temperature change.

$$\Delta Q = cm\Delta T$$

where ΔQ is the heat transferred, c is the specific heat, m the mass of the object and ΔT is the temperature change. Specific heat, c is the heat required to raise a unit mass of a substance a unit degree and will make the units of the above equation work out. (For example, c can be measured in joules / kg ˚C.) It probably should be mentioned that heat capacity is the total amount of heat required to change an object a unit temperature change and specific heat is the heat required to change a unit mass of an object a unit temperature change. (Often in Chemistry the molar specific heat is used and has the obvious meaning of the heat required to raise a mole of a substance a degree temperature change.)

Temperature remains constant during a phase change so the above relationship does not apply during phase changes. Below is a graph for taking ice from -10˚ C to steam at 110˚C. Students should be able to appreciate these familiar phase changes for water.

The graph on the right is the temperature vs. heat change when one gram of water is heated from -10˚C to



In the graph above, a gram of ice at -10˚C is heated to 0˚C, melted to water requiring 80 calories, is then heated to 100˚C, and boiled to steam requiring 540 calories and finally heated a little more after boiling all to steam. The specific heat of ice and steam are each about 0.5 calories/gram˚C. The term "latent heat" is applied to the amount of heat required to cause a phase change. The latent heat to change ice to water is 80cal/gram and the latent heat required to change water to steam is 540 cal/gram (both at atmospheric pressure.)

Thermal expansion

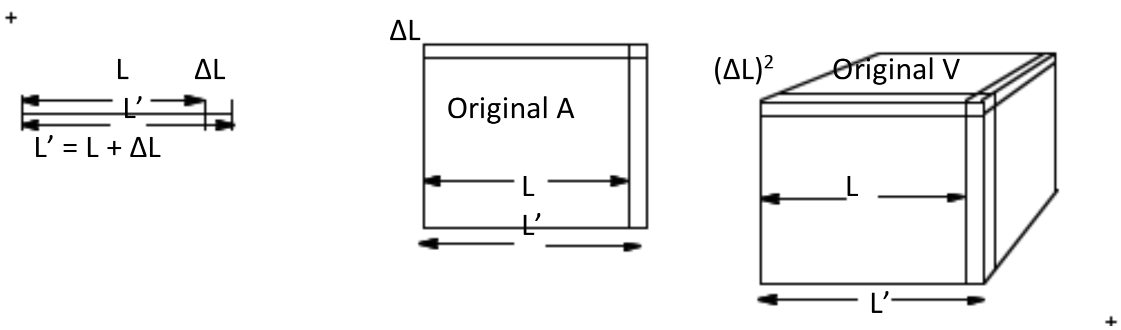
That objects expand when heated is well understood and discussing linear, area and volume expansion should be quite accessible to students. Linear expansion can be expressed:

$$L' = L + \Delta L \quad \text{and} \quad \Delta L = \alpha L \Delta T$$

Where L' is the new length after the temperature change ΔT, L is the original length, and α is the coefficient of linear expansion, usually a very small number in units of per unit temperature change. Area expansion can be expressed $A' = A + \Delta A$. When area can be expressed as L^2, the expanded length expression can be squared and when the very small values of α^2 are ignored the expression for area expansion becomes $A' = A + 2\alpha A\Delta T$ where the coefficient of area expansion $\beta = 2\alpha$. This operation can be repeated for volume expansion with $V' = V + \Delta V$. By cubing the length expression and dropping out the higher powers of α. The coefficient of volume expansion $\gamma = 3\alpha$. The final expression for volume expression: $V' = V + 3\alpha V\Delta T$

Liquids behave in the same way as solids but gasses should be treated with the gas laws (discussed below) since <u>pressure</u> as well and temperature becomes much more important.

The algebra suggested in the above discussion can be visually demonstrated as follows:



Note in the above illustration that when the area expands by ΔL on a side, the area increases by $2\ \Delta L\ L$ and this is ΔA. Hence $A' = A + \Delta A = A + 2\alpha L^2\Delta T = A + 2\alpha A\Delta T$
With the volume expansion, note that there are 3 expanded volumes each $\Delta L^2 L$ and it will follow using the same argument as with area $V' = V + \Delta V$ will lead to $V' = V + 3\alpha V\Delta T$

A wonderful question for students to ponder is: What happens to a small hole in a large metal plate when it is heated in an oven? Does the hole get larger or smaller? Several ways of answering the problem are possible and even the famous ring and ball demonstration might help but something about the plate being large and the hole small presents conceptual problems. One thing <u>not</u> to do is to go into a donut shop and ask them for an experimental answer. Apparently, what they report for donut holes in the dough before and after cooking does not correspond to what should happen to a large metal plate with a small hole in it. (At least that's what I'm told.)

Work, heat and the Joule mechanical equivalent of heat experiment

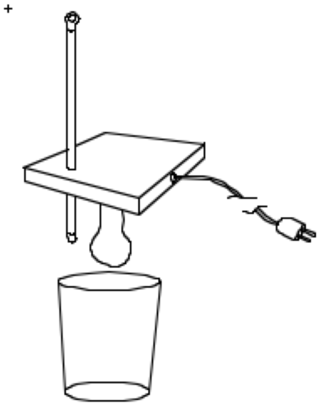
Historically the question of the relationship between mechanical work and heat had been debated for a long time before Joule did the definitive experiment. The details can be found in almost any physics text but the basic idea is allowing falling weights to fall at constant velocity while attached through a string and pulley arrangement to a special paddle wheel inside of a water filled calorimeter. A known amount of work can then be equated to a known amount of heat. The important result that should probably be learned is approximately:

4.185 joules = 1 calorie, or 4185 joules = 1 kilocalorie.

Joule also did the experiment using resistors in water and supplying a known voltage and resulting current to determine the amount of "work" but at the time electrical measurements were not as reproduceable as mechanical measurements.

An experiment that can be done using a light bulb in a Styrofoam cup can reproduce Joule's electrical experiment.

The illustration on the left shows the basic apparatus. A light bulb is inserted in a square of Styrofoam and attached to wire to an electrical plug. A thermometer is also threaded through this square of Styrofoam near the bulb and the entire apparatus will be inserted into a Styrofoam cup filled with a measured mass of water. After the bulb and thermometer are completely immersed, the plug is inserted into an outlet for a measured amount of time. If the wattage of the bulb is known, the product of the wattage and time will give the Joules transferred to the water and the mass and temperature change of the water will give the calories. (The equipment for this experiment is given to NPTW participants every other year.) This experiment always has about 10% error that can be easily explained!



(The Styrofoam cup is translucent and the light that the bulb is supposed to produce escapes.)

The Laws of Thermodynamics

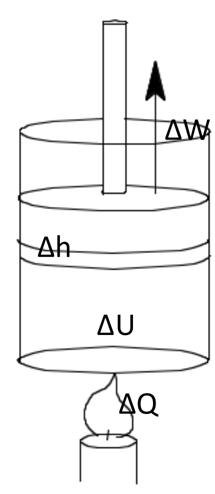
With today's very real concern about environmental problems, the laws of thermodynamics definitely should be discussed. The first law of thermodynamics discusses how the internal energy of a system is changed by the amount of work and heat going in and out of the system. The law is easy to state mathematically but can seem different in different discussions if it is not clear whether a plus or minus sign is to be used when the work or heat is entering or leaving the system. In this discussion we will use a + for heat or work going into the system and a – for heat or work leaving the system. (Should you see a different statement in other texts, study carefully what the + or – means.)

First Law of Thermodynamics: $\Delta U = \Delta Q + \Delta W$

Where ΔU is the change in internal energy of the system, ΔQ is the heat transferred into the system and ΔW is the work done on the system. Naturally, all units of energy and work must be the same.

The following is a very simple example of an application of the first law of thermodynamics in an idealized system that will be applied later to heat engines:

The illustration on the left shows a cylinder with a volume of air trapped in it by a piston of mass, m, that confines the air and presses downward on the air. Heat ΔQ is applied to the bottom of the cylinder causing the piston to rise up a distance Δh. As a result, the work done on the rising piston: $\Delta W = mg\Delta h$. Applying the first law of thermodynamics, the increase in internal energy of the system is: $\Delta U = \Delta Q - \Delta W$ (since ΔW out) $= \Delta Q - mg\Delta h$. The increase in internal energy will probably occur by increasing the temperature (or kinetic energy of air molecules) confined by the piston. Since the piston is allowed to rise freely, there will be no increase in pressure.



The <u>second law of thermodynamics</u> identifies the direction of natural heat flow:

Heat flows (naturally) from a high temperature source to a low temperature source.

There are many different ways of stating the second law of thermodynamics, often involving the concept of entropy. The above statement probably will make more sense for a first-time presentation. The second law of thermodynamics is consistent with our relating heat to the random kinetic energy of molecules and could be used to clarify why we wear jackets on cold days. Not to keep the cold from penetrating into us but to keep the internal energy (heat) of our bodies from escaping.

The ideal gas laws

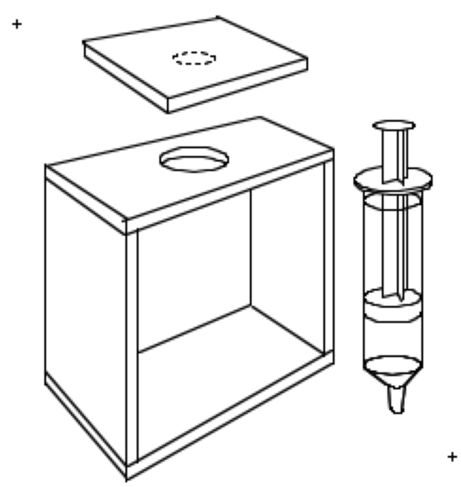
These have often been previously introduced in Chemistry. If they are new to your students, they deserve some attention.

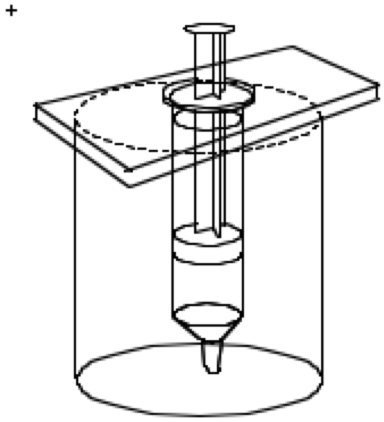
In the 17th and 18th centuries several scientists performed extensive investigations with gasses leading to what is now known as the ideal gas law. Boyle's law states that if temperature is held constant, the pressure times the volume of a confined gas will remain constant, or $PV = K$. Charles's law states that if pressure is held constant, the volume of a confined gas will be proportional to the absolute temperature, or $V = k'T$. (Graphs of these two laws are shown on the next page.)

The assorted gas laws can be combined to form the ideal gas law:

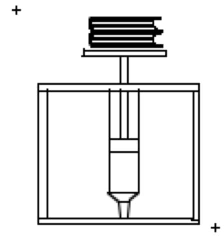
> $PV = nRT$ where pressure and volume can be any units but temperature must be kelvin. n can be the number of molecules or the number of moles of the gas and R is selected to make the units work. A popular value of R is 8.31 J $mole^{-1}$ $(^{\circ}K)^{-1}$ and is called the universal gas constant.

Doing gas law experiments are not complicated but they do take some lab time and perhaps special lab equipment. On the left is illustrated apparatus that can be purchased fairly inexpensively or even constructed. Large syringes of 100ml to 150ml can be purchased quite inexpensively and the structure shown is constructed to hold the syringe firmly with the board shown on top to hold weights or books. The lower end of the syringe is sealed (a cutoff pencil erasure can act as a stopper) and the sealed enclosure below the piston can be compressed with weights and the inside volume measured. A caliper will be required to measure the inside diameter of the syringe.

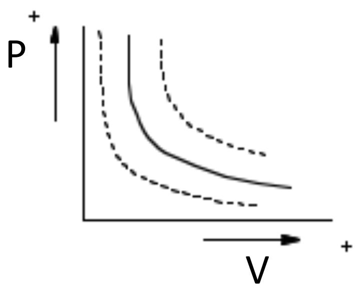




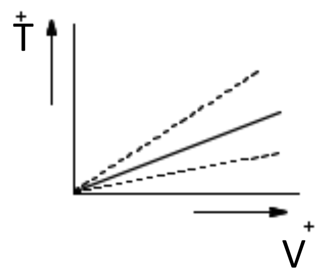
If a single board like the top board of the above structure is allowed to support the syringe, (shown on left) it can be lowered into several different large containers with different temperature water (ice, room. higher) to obtain constant pressure at different temperature for Charles' Law.



Above is illustrated the use of books to vary the pressure of the apparatus.

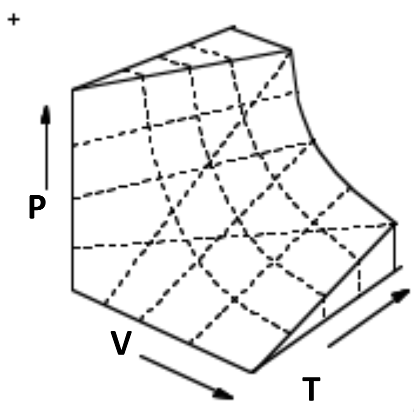


On the left is the pressure vs. volume graph for Boyle's law and on the right is the temperature vs. volume graph for Charles' law. The temperature vs. volume graph is a great way to extrapolate from "normal" temperatures to absolute zero.



The Charles's law graph can be extrapolated to determine absolute zero. Obviously, no real gas would be able to go all the way to absolute zero as might be suggested in the temperature vs. volume graph above. However, many gasses at "normal" temperatures obey the ideal gas law very closely.

Since the ideal gas law, PV = nRT involves three variables, a graph of all three variables would involve three dimensions or a surface. The illustration on the left shows all three variables of pressure, volume and temperature on a single surface. It is possible to view the two-dimensional graphs by looking along the appropriate axis and through the appropriate face. For example, Boyle's law is seen by looking across the PV face in the direction of the T axis.



Pressure vs. Volume graphs and heat engines

Pressure vs. Volume graphs are very helpful when learning about the thermodynamics of basic heat engines. The application of thermodynamics to automobile engines, heat pumps, refrigerators and many other devices that must be considered in light of their environmental consequences are both relevant and interesting to students.

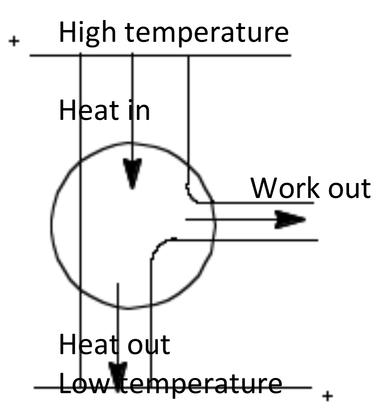
In the following discussions we will initially confine the discussions of heat engines to a simple cylinder and piston in which heat enters and pushes up the piston as discussed above in the **Laws of Thermodynamics** on page 19.

From the definition of pressure: $P = \text{Force/Area} = F/A$ and since the cross-sectional area A of the cylinder is constant, we can say the volume swept out by the moving piston is $\Delta V = \Delta h\ A$. This means that pressure times volume change or $P\Delta V = (F/A)X(\Delta h\ A)$ or $P\Delta V = F\Delta h$ But, this is the work done in moving the piston up a distance Δh. This can be generalized into an important result:

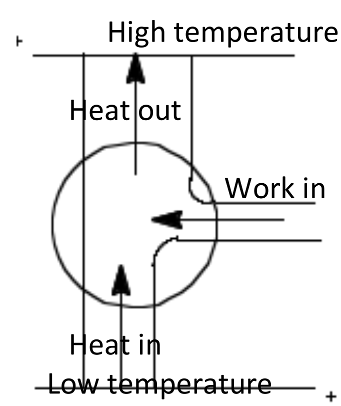
The work done in any action on a confined volume of gas is the <u>area under a pressure vs. volume graph.</u>

Since we learned from the Joule mechanical equivalent of heat experiment that work and heat are equivalent (of course for calculations, the units must be the same) we can discuss the heat and work that goes in and out of a gas changing volume with a piston above it by drawing a pressure vs. volume graph or a PV graph.

The first heat engine was not invented by James Watt but he was the first to appreciate that all heat engines must rely on heat being transferred from a high temperature source to a low temperature source (2nd Law of Thermodynamics) and that the work extracted from this heat transfer would be the difference between the heat in from the high temperature source, minus the heat transferred to the low temperature source. (1st Law of Thermodynamics). Watt's innovation was to separate the high temperature source from the low temperature source.



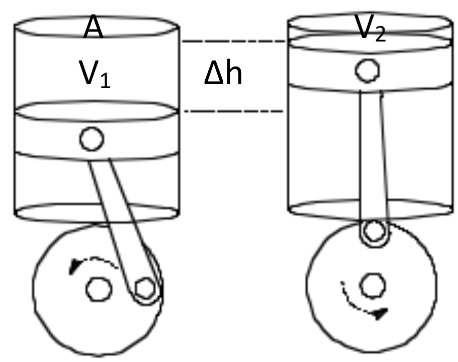
Typical schematic diagrams of heat engines (shown on the left) and heat pumps or refrigerators (shown on the right.) The 1st Law of thermodynamics describes the heat and work in and out of the system and the 2nd Law describes the direction of heat flow. If the refrigerator seems to violate the 2nd Law, remember that the work <u>in</u> also requires some sort of engine that will involve <u>more,</u> high to low heat transfer than the reverse by the refrigerator.

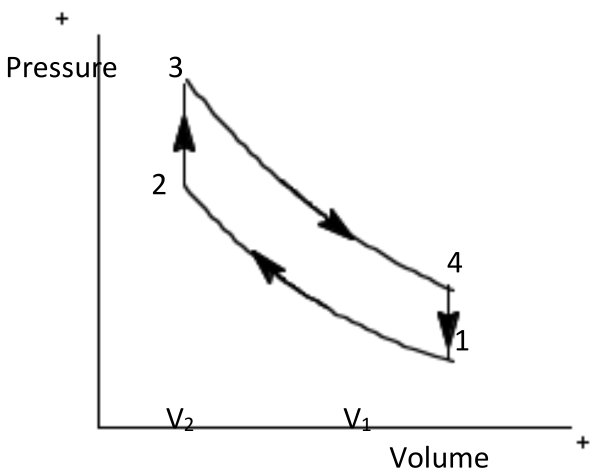


The circles in the diagrams above represent the systems using heat to produce work (heat engine) or using work to reverse the direction of heat flow (heat pump or refrigerator). More detailed PV diagrams can help to understand what is happening in actual systems. (A discussion of the typical internal combustion engine used in cars will be discussed on the next page.)

To begin a brief discussion of how modern heat engines work, we will first consider a piston that can move up and down in a closed cylinder:

The illustration on the right represents a simplified piston engine that is closed on the top. The crossectional area of the cylinder is A, and the piston moves up a distance Δh. The initial volume above the piston is V_1 and the final volume is V_2 making $\Delta V = V_2 - V_1$. A pressure volume graph will represent the amount of work done to compress the gas above the piston. (We will ignore the weight of the piston, friction, etc.) In a subsequent discussion we will revise this very simple engine to show what goes on in real engines.





The pressure-volume graph on the left represents the work done to compress the gas in the piston above. The path from 1 to 2 represents the compression from V_1 to V_2 and the area under this curved line down to the volume axis would represent the work done to accomplish this compression. At point 2, if something were done to rapidly heat the gas above the piston, the pressure would rise to point 3 on the graph. Now if the piston moves downward to point 4 on the graph, more work would come out from the engine than was done to compress the gas. If at point 4 something was done to cool the gas to its original temperature, the cycle could begin again.

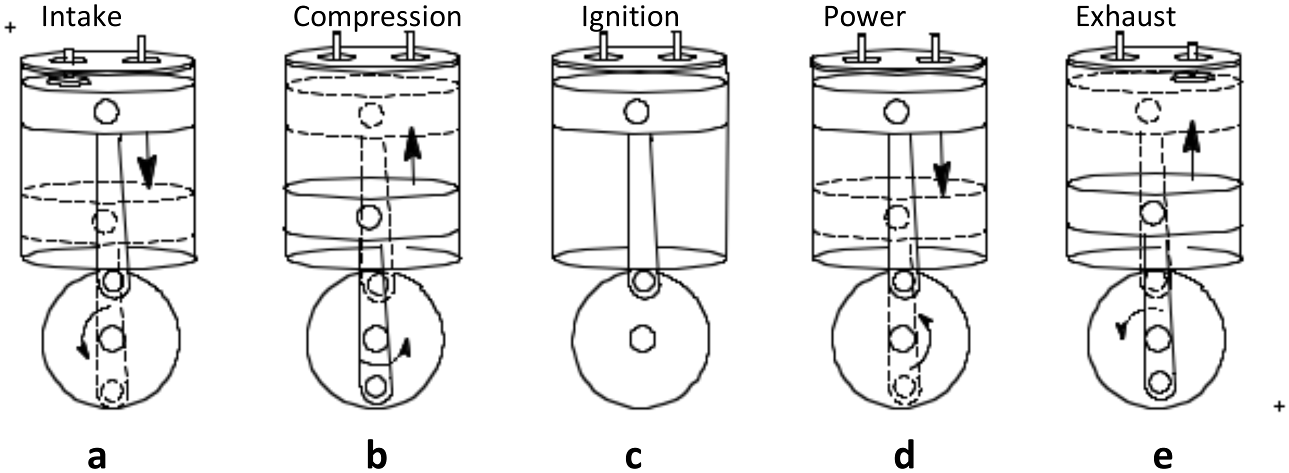
To get an idea on how the above discussion applies to a real engine we will now discuss the essentials of a 4-stroke internal combustion engine that is commonly used in most cars and small trucks. Many excellent vides of this type of engine can be found on the web. Consider:

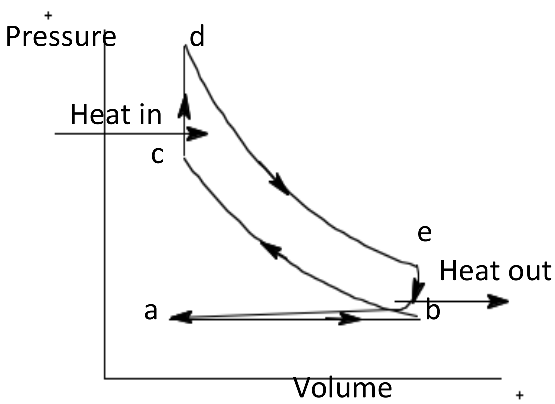
A great discussion of a single cylinder 4 stroke engine:
http://www.youtube.com/watch?v=Pug73uIG6Zo

Another discussion of the typical 4-cylinder engine used in many small cars. This discussion devotes the first part of the video to an animation of how the engine is assembled that would help students to come to understand all the parts of the engine. The final part of the video shows all 4 cylinders in operation:
http://www.youtube.com/watch?v=BXQ27pU37E

If this does not work, simply type "4 stroke engine" into your browser and many will appear.

In the illustrations below, the intake valve is on the left and the exhaust valve is on the right.





Steps illustrated on the PV graph on the left:

a –b Intake stroke. Fuel air mixture is drawn into cylinder. Intake valve open and exhaust valve closed.

b—c Compression stroke. Fuel air mixture is compressed.

c—d Ignition. At the top of the compression stroke, the piston hardly moves as the fuel air mixture is ignited.

d—e Power stroke. At the increased pressure and temperature, the piston moves down and does work.

e—a Exhaust stroke. The piston moves up with the exhaust valve open and intake valve closed. Heat leaves system to low temperature source.

In steps b, c and d both valves are closed.

The two-stroke engine used in small engines on garden tools and go-carts, the piston acts to open and close the intake and exhaust ports. An excellent video can be found at: http://www.youtube.com/watch?v=Z6YC3154so4

The concept of the Carnot engine and engine efficiency may be too advanced for introductory High School Physics but excellent discussions can be found in any University Physics Text or by seeking "Carnot Engine" on the web. Two brief points will be made here:

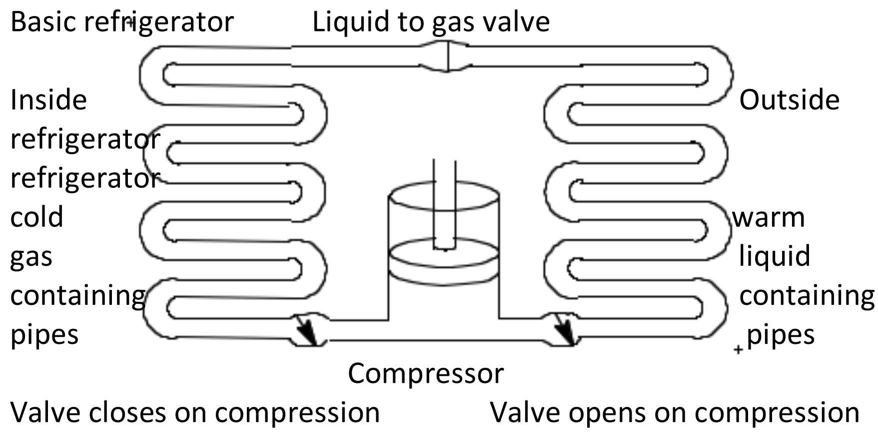
1. Engine efficiency increases if the difference in temperature between the high and low temperature source increases.
2. The more nearly an engine is "reversible" the higher will be its efficiency.

When considering the use of energy to heat homes, it might be useful to consider the total amount of energy used with a typical furnace compared with a heat pump. A furnace begins with a high temperature flame that must be cooled before entering the house. A heat pump does a much smaller amount of work on nearly the same temperature air and brings it to the temperature of the house. The discussion of the refrigerator below helps explain this.

Reversibility and refrigerators (also air conditioners and heat pumps).

A discussion of how a refrigerator or heat pump works would be appropriate for a High School class. Its basic operation is important to understand when considering problems related to energy conservation and global warming.

In the basic refrigerator diagram on the right the fluid in the coils moves counterclockwise. The compressor converts the gas moving in from the left to a liquid on the right. This liquid is warmer than room temperature and radiates heat out into the room. At the top, the liquid to gas valve allows the liquid to evaporate, cooling it to a much lower temperature inside the refrigerator



The refrigerator transfers heat from a low temperature source to a high temperature source in seeming violation of the 2nd Law of Thermodynamics. But the work required to run the compressor comes from some sort of engine that requires more heat transfer from a high temperature source to a low temperature source, the overall effect is no violation. All refrigerators and heat pumps use a working fluid that changes from a liquid to a gas at near room temperature. The evaporation of the liquid at the liquid to gas "throttling" valve results in a large temperature drop during the phase change.

Reversibility and engine efficiency

A perfect heat engine would convert all the heat coming into it into useful work. This is impossible! In the early stages of heat engine development, the question was: "What engine will convert the greatest amount of heat to work?" In 1824 Sadi Carnot published a paper describing the maximum efficiency heat engine operating between two given temperatures. Details of the argument are involved but the key idea is that no engine can be more efficient than a reversible engine. A simple argument should help to understand why this is true:

The illustration on the right shows a reversible engine used as a refrigerator being driven by a hypothetical super engine that is more efficient than the reversible engine. Since the super engine is more efficient than the reversible engine, the difference between the heat out of the high temperature source transferred to the low temperature source must be less than the equal amount of work required to drive the refrigerator. This means that more heat will be transferred to the high temperature source than to the low temperature source! This violates the 2nd law of thermodynamics. Therefore, the super engine is impossible.