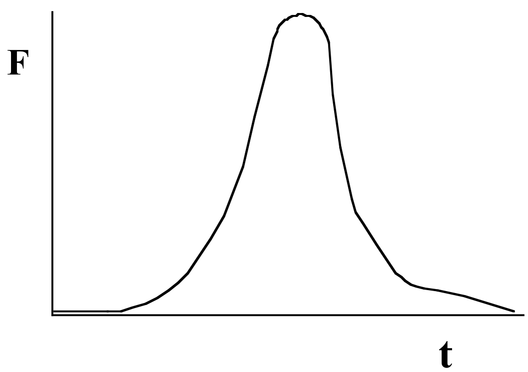
Using the collision carts and track to demonstrate in-line collisions.

An earlier paper describes how to construct a track and magnet carts for the purpose of demonstrating the energy and momentum transfer during in line elastic and nonelastic collisions.[1] The paper below suggests how to make the best use of this apparatus with a high school physics class. It is assumed that Newton's second law has been introduced and can be use to develop the impulse momentum theorem. From

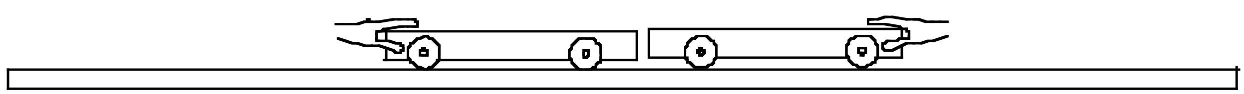
$$F = ma = m\,\Delta v/\Delta t \text{ which can be rearranged to read } \mathbf{F\,\Delta t = m\,\Delta v = \Delta\,(mv)}$$

That is, impulse equals change in momentum. It is often useful to extend the definition of impulse to the area under a force time curve. This allows discussing changing forces over extended periods of time. This is particularly useful during collisions since here the force is rarely constant. During a collision between two carts, even though the exact shape of the impulse graph can not be specified, it can be asserted that the impulse graph for one of the carts will be exactly the same, only with the force in the opposite direction, as the impulse graph on the other cart. This means that the change in momentum of one of the two carts must always equal the change in momentum of the other cart, only in the opposite direction. (Here we assume the friction of the track is negligible.)

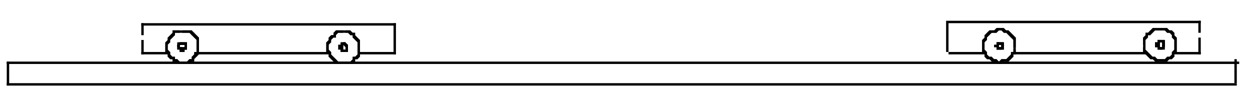


The physics of simple explosions

Start the discussion of collisions by holding the two carts as close as you can with the magnets opposing your force and releasing each cart as carefully as you can at the same time.



After release, the carts will move about the same distance apart following the "explosion". Begin the discussion by asking: "How did the force on each cart compare while they were moving apart?"

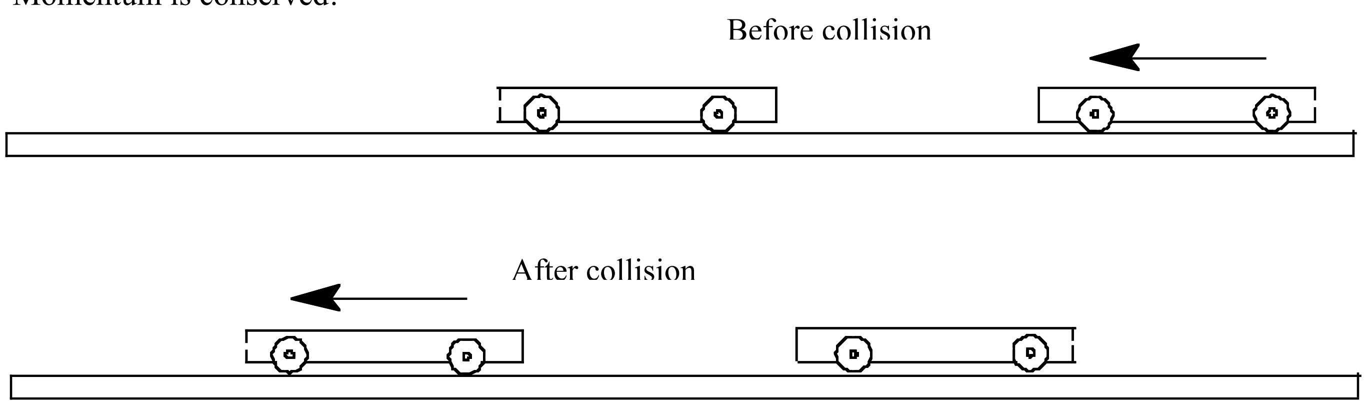


(Newton's third law should help here.) Now ask: "How does the time that the cart on the left experiences this force compared with the time the cart on the right experiences the force?" (Simple logic but some may have trouble with this.) After you establish that each cart experiences the same magnitude force in the opposite direction for the same length of time, it must follow that the change in momentum of each cart must have been of the same magnitude only in the opposite direction. Repeat this demonstration only with one of the two carts loaded. The large mass cart will move a smaller distance. Argue that during the explosion the impulse each cart received was still of the same magnitude only in opposite direction therefore the magnitude of the momentum change of each cart must have been the same. Therefore the larger mass cart moved at a smaller velocity, conserving momentum. (This is probably a good time to begin repeating the generality of the conservation of momentum principle.)

The physics of elastic and nonelastic collisions

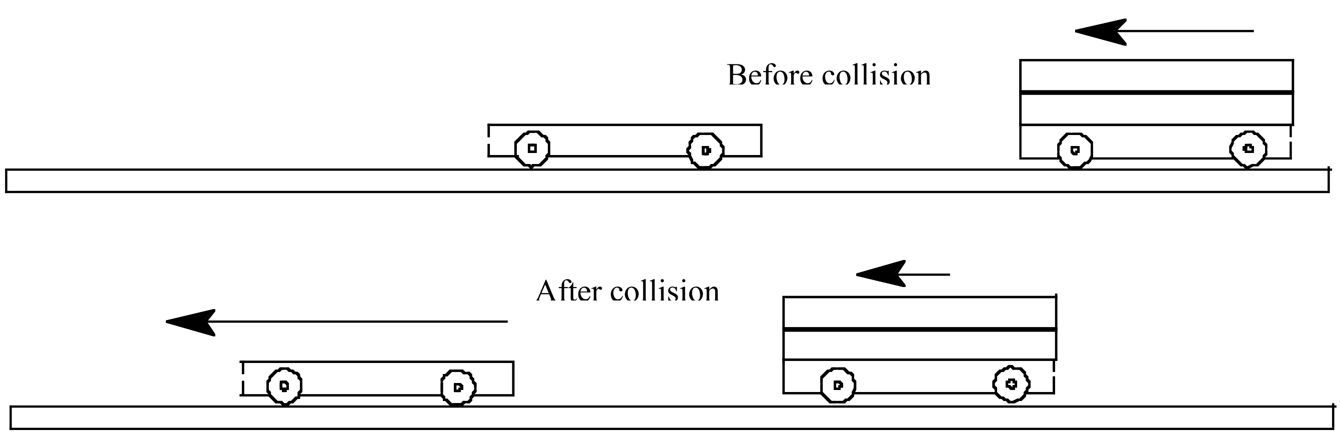
(Notice that we use the word “nonelastic” rather than “inelastic”. This helps to avoid the confusion that can come up when trying to distinguish: “in elastic collisions” from “inelastic collisions”.) Students are usually amazed to see the carts collide under repelling magnetic forces but help them to understand that even the quick collisions between elastic steel balls (for example with the Newton’s Cradle) build up to a maximum force and fall back down to zero in a finite time. The magnetic repulsion forces only make it easier to see this happen.

Begin the demonstration with a simple elastic collision between a moving cart and a stationary cart. The students should observe that the moving cart comes to a complete stop and the stationary cart now moves off with almost the same velocity as the originally moving cart. Momentum is conserved!



Now reverse the direction and the same result will be observed. A subtle point to make at this time might be to observe that the moving cart will probably bounce back slightly after the collision. This will happen in either direction if the carts are very nearly of the same mass and the track is level. The explanation for this is that the track is not frictionless and some of the momentum of the moving cart during the collision is transferred to the ground through the track hence requiring the moving cart to bounce back to conserve momentum.

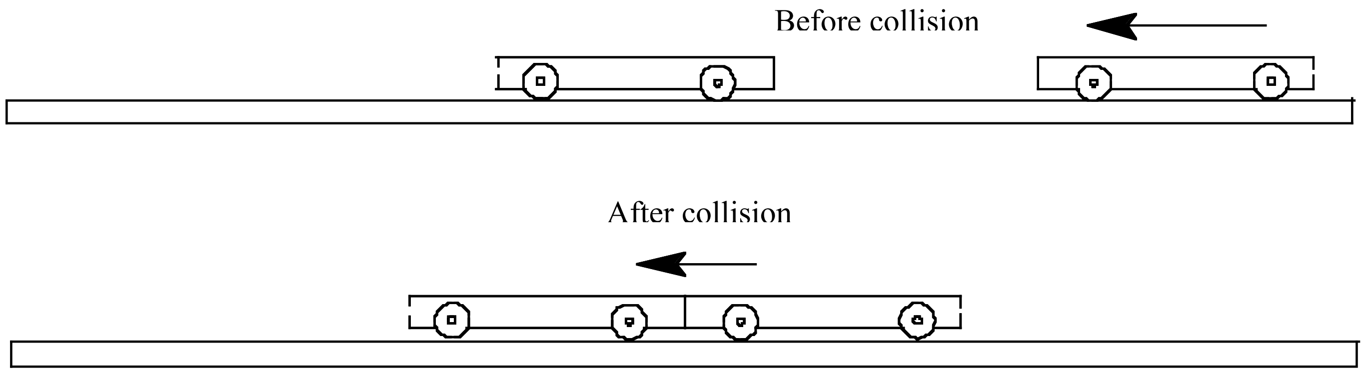
Now load the moving cart and have the students predict the resulting collision.



A qualitative discussion should lead to the conclusion that the loaded cart will not be able to "unload" all of its momentum before it comes to rest, hence must continue to move after the collision at a smaller velocity. The unloaded cart will move off at a much higher velocity.

Again, reverse the direction and have the students discuss the result. Here the unloaded cart must bounce back since it will still be interacting after it comes to rest. Discuss qualitatively how momentum is conserved in each case.

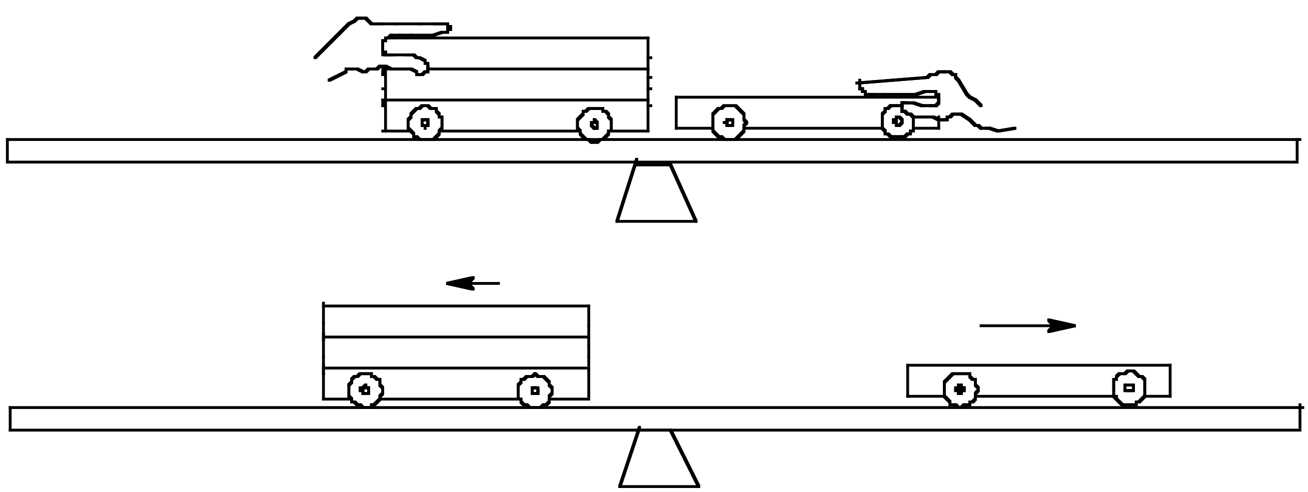
Now turn the carts around on the track so the Velcro on the other end will produce nonelastic collisions. It is fairly easy to see that with equal mass carts, when they move off together, the velocity of the combined carts is about half the velocity of the original moving cart.



Repeat these nonelastic collisions with unequal and equal loaded carts and discuss the results.

The motion of the center of mass of a system before and after explosions and collisions

An important consequence of the conservation of momentum principle is that the motion of the center of mass of a system before, during and after any kind of interaction will remain unchanged. This means if we can identify the motion of a rocket-fuel system before firing, the motion of the center of mass between rocket in one direction and the fuel in the other direction will be exactly the same as it was before the rocket fired. Excellent illustrations of this can be found in the PSSC text .[2] A crude demonstration of this can be done with the collisions carts and track. Carefully balance the track with a loaded and unloaded cart with the repelling magnets facing one another.



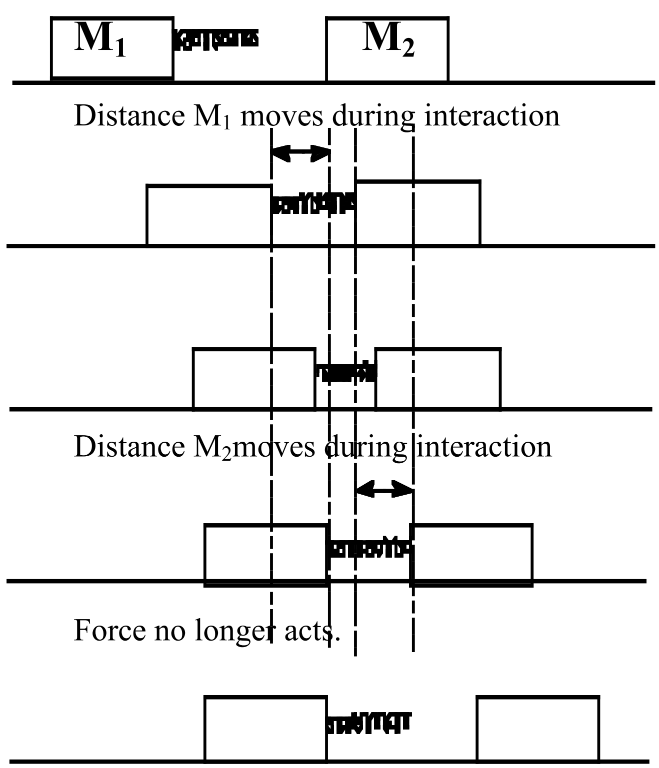
The balance fulcrum should probably be flat on top since a knife-edge critical balance is difficult to maintain. First hold the two carts near one another against the magnet repulsion and then release them at exactly the same time. If the release was properly done, the carts should move off at different speeds yet the track should remain in balance as they move away, illustrating that the center of mass of the system did not move. This demonstration is difficult to perform and may only serve as a focus for discussion. Since releasing each cart at exactly the same time is difficult, the demonstration might be improved if they are tied together with a light string, balanced, followed by burning the string. Success or failure, this demonstration can initiate a discussion of how the center of mass of an even complex system like an exploding shell will have the same motion of its center of mass before, during and after the explosion. Often collisions, as well as explosions, can be made easier to analyze using a center of mass coordinate system.

Conservation of energy and momentum during collisions and explosions

The above discussions have concentrated on conservation of momentum. The conservation of energy principle is also involved. The following discussion helps to use the idea of work and energy during an elastic collision to show how energy is exchanged:

It is important to stress that before and after elastic collisions, kinetic energy as well as momentum is conserved. The forces that act between the colliding bodies are conservative and will return all of the energy that was temporarily stored when the bodies were at their closest distance separation. (Another way of saying a spring or magnet interaction is "perfectly elastic" is to say the force depends only upon separation. Springs are elastic since their restoring force depends only on the distance they are compressed or stretched. Balls of putty exert one force when they are compressed but do not return this force and do not elastically expand again.) Hopefully students will appreciate the following argument involving work to show that the kinetic energy that is temporarily stored as potential energy when the objects are at minimum separation will all be returned if the bumpers are perfectly elastic.

The important point to realize is that if the spring is perfectly elastic, the force applied on M_1 while it slows down will always be equal and opposite to the force that speeds up M_2 for the entire distance of the complete interaction. Since force and distance represent work, the work "done" in slowing M_1 exactly equals the work done in speeding up M_2. The kinetic energy lost by M_1 is transferred to M_2. If, on the other hand, the spring had been inelastic (say a coil of soft solder) it would not have returned the energy after compression and more energy would have been lost by M_1 than would have been gained by M_2.



Momentum is always conserved since the force and time are the same no matter how far each object moves during the interaction. The elastic collision transfers both momentum and kinetic energy. The inelastic collision also observes momentum conservation but not kinetic energy conservation. It is also interesting to note that during the heart of the elastic collision, even kinetic energy is not conserved since some energy is temporarily stored in the spring as potential energy.

Solving elastic and inelastic collision problems.

(The algebra discussed below may not be appropriate for some physics classes, particularly if physics is taught in earlier grades.)

Solving nonelastic collision problems in one dimension is fairly straightforward since the two objects stick together after the collision, therefore move at the same velocity. A simple example would be:

Find the final velocity if a mass m_1 moving at velocity v_1 collides nonelastically with a mass m_2 moving at velocity v_2.

Since they stick together, we apply momentum conservation:

$m_1v_1 + m_2v_2 = (m_1 + m_2)\, v'$ and solving for v' is simple algebra.

However, if the same problem is said to be a perfectly elastic collision, the masses will be moving at different velocities after the collision and will involve solving two equations in two unknowns. Although this is not impossible, it will involve a little messy algebra. We will do this solution here because the final result is interesting and useful. Again, mass m_1 moving at velocity v_1 collides with mass m_2 moving at velocity v_2 and the collision is perfectly elastic. The problem is to find the final velocity v_1' of mass m_1 and the velocity v_2' of mass m_2 after the collision.

From KE conservation: $1/2m_1(v_1)^2 + 1/2m_2(v_2)^2 = 1/2m_1(v_1')^2 + 1/2m_2(v_2')^2$ Eq. (1)

and from momentum conservation $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$ Eq. (2)

Rearrange equation (1) so all terms in m_1 are on one side of the equation and m_2 on the other:

$1/2m_1(v_1)^2 - 1/2m_1(v_1')^2 = 1/2m_2(v_2')^2 - 1/2m_2(v_2)^2$

Dividing both sides by ½ and factoring m_1 and m_2 : $m_1[v_1^2 - (v_1')^2] = m_2[(v_2')^2 - v_2^2]$

From the difference of two squares: $m_1 (v_1 + v_1')(v_1 - v_1') = m_2(v_2' + v_2)(v_2' - v_2)$ Eq. (3)

Rearranging Eq. (2) $m_1v_1 - m_1v_1' = m_2v_2' - m_2v_2$ or: $m_1(v_1 - v_1') = m_2(v_2' - v_2)$ Eq. (4)

Dividing equation (3) by equation (4) yields: $v_1 + v_1' = v_2' + v_2$

Rearranging this equation gives an interesting result: $\boxed{\mathbf{v_1 - v_2 = v_2' - v_1'}}$ Eq. (5)

This is not yet the solution to the problem but it says something very important. This equation says the relative velocity before the collision equals minus the relative velocity after the collision. This is always true in all perfectly elastic collisions. If the baseball is perfectly elastic, the velocity the ball and bat approach one another before collision will always equal the velocity of separation after the collision. (This may be why long ball hitters prefer a fast ball pitcher.) A "super" ball should bounce up with the same speed it had just before striking the floor. This equation also makes it easy to solve the original problem. Now with equation (2) and equation (5) we have two simple linear equations that can be easily solved simultaneously for v_1' and v_2'

Perfectly elastic problem can be a mess to solve using simultaneous solutions of kinetic energy and momentum equations, but the result that the velocity of approach before the collision must equal the velocity of separation after the collision can make these problems much easier to solve.

Notes

1. Should you wish a copy of this paper, e-mail Bill Layton layton@physics.ucla.edu

2. Although most texts will include illustrations to help explain this point, any one of the many editions of the Physical Science Study Committee (PSSC) Physics Text will have excellent drawings and photographs stressing the use of the center of mass coordinate system. In the 4th edition, check chapter 14, section 14.6 "The Center of Mass" and section 14.7, "The Center-of-Mass Frame of Reference". Earlier and later editions may have the same sections. In any event, it should be found in the chapter called "Momentum and the Conservation of Momentum".